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Mechanical Perspective on the Rotational Doppler Shift of a Single Photon

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Author's contribution

Author RL has conducted the study and written the manuscript.

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ABSTRACT

A theoretical model that relates the rotational Doppler shift of a photon and the rotational velocity of the lenses traversed by the beam of light, is presented. The mathematical relation, which is resolved in the context of a four-dimensional Minkowski flat spacetime and Cartesian coordinates, relates the rotational Doppler effect of a circularly polarized electromagnetic wave, caused by the transfer of spin angular momentum from a rotating object (lenses), with the coordinate acceleration of the rotating object, in the counter-propagating direction in which the photons move, and its angular velocity. From the analysis of the solved equation, it can be considered the generated coordinate acceleration and the theoretical possibility that it was obtained from the mechanical energy of a rotating object traversed by a beam of light, which, in turn, would generate a coordinate acceleration difference in the parallel and counter-propagating direction in which the photons move.

Keywords: Spin angular momentum; coordinate acceleration; photon; rotational doppler effect.

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1. INTRODUCTION

The photon, the elementary particle responsible for the quantum manifestations of electromagnetism, has energy proportional to its frequency, as established by the Planck relationship [1,2].

On the other hand, it's known that.. "the gravitational field of a laser pulse has been investigated, showing similarities with Newtonian gravity in two dimensions: massless test particles do not experience any physical effect if they are co-propagating with the pulse, but they are accelerated if counter-propagating with respect to the pulse" (Rätzel et al.) [3].

It has also been shown that the rotation of polarized light beams with angular momentum (AM) (occurring naturally or created using spiral phase plates, spatial light modulators, or q-plates), including spin angular momentum (SAM) and orbital angular momentum (OAM), results in a frequency shift of photons [4,5], called the rotational Doppler shift (RDS) [6].

The frequency shift of a polarized beam of light, rotating with an angular velocity with respect to the observer, produces a frequency shift, which could be induced by rotating lenses in purely optical experiments [7-13].

In view of this phenomenon, an equation for this relationship is derived here based on a theoretical model that relates coordinate acceleration (a_g), in the counter-propagating direction of the light beam, to the rotational Doppler shift of a photon, as obtained by transfer of SAM from a rotating object to a circularly polarized beam of light passing through it.

2. METHODOLOGY

To simplify calculations, the following assumptions and analysis were implemented.

First, because a flat spacetime (ST) constitutes a good description of physical systems over finite distances in systems without significant gravitation, even in curved space, Minkowski ST is still a good description of an infinitesimal region surrounding any point. Indeed, Cartesian coordinates can be used in the infinitesimal neighborhood of a given point, which is considered as a small region of flat ST that is tangent to the curved ST at that point; in that region, a geodetic line of the world is a straight line. For this reason, local inertial frames can also be introduced into curved ST.

According to the flat Minkowski's space of four dimensions and for infinitesimally small distance scales, as for the single photon considered in this study, this space has been used. This flat topological space locally resembles Euclidean space near each point.

Thus, a photon was considered to be a point on this tangent space, which was described in this study by only two of its four Cartesian coordinates (one temporal-*x* and one spatial-*t*), to simplify the calculations.

Second, the equation of an electromagnetic wave (EM) in a plane [13,14] in the direction of the *x*-axis along the beam of light was assumed to be described by

$$\frac{\partial^2 \varepsilon(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \varepsilon(x,t)}{\partial t^2}, \tag{1}$$

where ε = electric field strength of a photon, c = speed of light, t = time coordinate, and x = x-component of Cartesian coordinates.

Third, for particles, such as photons that move at the speed of light (c), the ST interval (s), as the length of time or the distance between two events or positions in spacetime, is zero. Thus, the ST interval between two events on the world line (path tracing an object in a four-dimensional spacetime) of something moving at the speed of light is zero (i.e., $ds^2 = 0$).

Therefore, in a flat Minkowski space of four dimensions [15], Two of the four Cartesian coordinates (i.e., one -*x*-spatial and one -*t*-temporal) without loss of generality. Thus, c can be written as a constant function of dx and dt ($c=f(dx,dt)$)

$$c = \frac{dx}{dt} \tag{2}$$

.

Fourth, Planck's equation was also applied [1,2] as

$$E = h\nu, \tag{3}$$

where E = energy, h = Planck's constant, and υ = frequency. Then

$$\frac{dE}{E} = \frac{d\nu}{\nu} \tag{4}$$

.

Fifth and finally, the coordinate acceleration, which is defined as the derivative of the coordinate velocity with respect to the coordinate time, is zero when the trajectory is a straight line or non-zero when the trajectory is curved in a flat Minkowski.

The equation for the curve trajectory, written in Cartesian coordinates, wherein each point represents a photon, as noted above, and is assumed as twice differentiable and $x = f(t)$, can be simplified as

$$\kappa = \frac{-a_g}{\left\{1+\left(\frac{dx}{dt}\right)^2\right\}^{3/2}}, \tag{5}$$

where a_g coordinate acceleration = d^2x/dt^2 and κ = curvature since the first and second derivatives of y with respect to t were assumed to be 1 and 0, respectively [16].

Based on the particular condition of the inertial frame of reference (i.e., the photon), such as those presented in gravity-free coordinate systems that move at the speed of light with origin at t = 0, then equation (1) admits, as one of the simplest solutions for real values of x, an equation derived from equation (4) as follows:

$$\varepsilon = \alpha\ e^{\beta(\frac{dx}{dt})} \tag{6}$$

where α and β = constant (and β <0, to avoid that the field can be infinite).

The second derivative of equation (6) with respect to x gives us the following equation:

$$\frac{\partial^2 \varepsilon}{\partial x^2} = \alpha\beta^2 \frac{1}{(dt)^2} e^{\beta(\frac{dx}{dt})} \tag{7}$$

This is obtained by considering that $\partial(dx)\ /\ \partial x = 1$ *and* $\partial(dt)\ /\ \partial x = 0$ according to the definition of partial derivative for *f(x) = dx = Δx→0* and *g(t) = dt = Δt→0*. (note, however, that the derivative of equation (2) is *d(dx/dt=c)=0*.

The second derivative of equation (6) with respect to t gives us the following equation:

$$\frac{\partial^2 \varepsilon}{\partial t^2} = \alpha\beta^2 \frac{c^4}{(dx)^2} e^{\beta c^2 (\frac{dt}{dx})} \tag{8}$$

This is obtained by considering that $\partial(dt)\ /\ \partial t = 1$ and $\partial(dx)\ /\ \partial t = 0$ according to the definition of partial derivative for *f(x) = dx = Δx→0* and *g(t) = dt = Δt→0*. (note, however, that the derivative of equation (2) is *d(dx/dt=c)=0*.

Now, equation (7) is divided by equation (8).

according to equations (1) and (4).

equation (6) it can be rewritten

$$d\varepsilon = \alpha\beta \frac{dx}{dt} e^{\beta(\frac{dt}{dx})} \tag{9}$$

.

Further, dividing equation (9) by equation (6), we obtain

$$\frac{d\varepsilon}{\varepsilon} = \beta \frac{dx}{dt}. \tag{10}$$

Likewise, the energy density η of the electric field, which in the case of circular polarization is constant [15], is given by

$$\eta = \frac{E}{V} = \frac{1}{2} \varepsilon_o \varepsilon^2 \tag{11}$$

and

$$E = V \frac{1}{2} \varepsilon_o \varepsilon^2 \tag{12}$$

where V = volume and ε_o = vacuum permittivity.

The derivative of E with respect to ε is

$$\frac{dE}{d\varepsilon} = V \varepsilon_o \varepsilon. \tag{13}$$

Therefore, dividing equation (13) by equation (12), we obtain

$$\frac{dE}{E} = 2\frac{d\varepsilon}{\varepsilon}. \tag{14}$$

Using (8) here, then

$$\frac{(d\varepsilon)^2}{\varepsilon^2} = \beta^2 \frac{1}{4}[(\frac{-a_g}{\kappa})^{2/3} - 1] \tag{15}$$

And

$$\frac{(dE)^2}{E^2} = \beta^2 [(\frac{-a_g}{\kappa})^{2/3} - 1] \tag{16}$$

.

Then, using (5) here:

$$\frac{d\nu}{\nu} = \beta[(\frac{-a_g}{\kappa})^{2/3} - 1]^{1/2} \tag{17}$$

.

3. RESULTS AND DISCUSSION

The obtained equation (17) relates the variation in the frequency of an EM to a_g. The theoretical possibility of generating changes in coordinate acceleration associated with the rotational Doppler shift of a light beam follows from this equation.

Having established the quantitative relationship between the rotational shift of a photon and the coordinate acceleration, it is important to identify suitable practical systems for transforming shifted light into coordinate acceleration. In this context, EMs have been known for decades to carry spin angular momentum (SAM) corresponding to either the left or right circular polarization states. As a consequence, a rotational change in frequency called the rotational or angular Doppler effect occurs, accompanied by transfer of energy from or toward the light [17]. This energy transfer is described as

$$d\upsilon = \pm\sigma\ \Omega \text{ (for } \Delta\upsilon\to 0\text{)}, \tag{18}$$

where Δv is the angular frequency shift for one photon of the incident light, Ω is the relative angular velocity, and $\sigma = \pm 1$ is the SAM [3]. Substituting the expression from equation (18) into equation (17), and after a reordering of terms and taking into account that $(\sigma = \pm 1)^2=1$, can be written for a number of photons n as the final result.

$$a_g = \sum_i a_{g_i} = -\sum_i K\left[1+\left(\frac{\Omega_i}{\beta\nu_i}\right)^2\right]^{3/2} \tag{19}$$

,

and

$$\Delta a_g = a_{g_{final}} - a_{g_{initial}} \tag{20}$$

where v is the frequency of incident light, and ag and Ω_i, are the x coordinate components of a_g and angular velocity, repectively, for each of the n photons; and Δag the difference in the coordinate acceleration before and after encountering the rotating object.

The rotational Doppler shift effect of light causes a small change in wavelength, on the order of a few nanometers, because the rotational speed of objects is small compared to the speed of light [18].

From equation (19), it can be concluded that change in frequency of one photon (or a_{gi} for n photons) is directly proportional to the AM transferred by the relative rotational movement of the traversed object (Ω) with respect to the circularly polarized light beam traversing it ($\sigma = \pm 1$) [19,20]. Therefore, the interaction of light rays with the material implies that large energy exchanges in the form of AM would produce a strong rotational Doppler effect [19,21].

Finally, noted since the magnitude of the Doppler change due to AM exchange is not theoretically defined for rotating objects, different research groups have carried out various experimental determinations for various applications. Such experimental control leads to increasingly accurate predictions of the magnitude of the exchange (e.g., the yield of the procedure) [20-22].

It is acknowledged that in order to simplify the calculations, the present study reduced the mathematical resolution of the problem to a flat ST of two dimensions, a single Cartesian coordinate (x), and constant curvature.

Therefore, the results obtained are not generalizable. However, we believe that despite these limits to its applicability, it opens a new line of research in the utilization of the rotational Doppler effect.

4. CONCLUSION

This study resolved the relationship between coordinate acceleration and the rotational Doppler shift effect of the circularly polarized electromagnetic waves caused by transfer of SAM from a rotating object. This analysis considered the generated coordinate acceleration and its theoretical possibility of obtained from the mechanical energy of a rotating object crossed by a beam of light. In this way, the mentioned rotational Doppler shift would generate a difference of coordinate acceleration, parallel and in the counter-propagating direction in which the photons move, due to the difference in the coordinate acceleration (Δag), derived of the frequency and energy of a light beam, before and after encountering the rotating object. This difference would generate thrust and could be used to drive the rotating object. In other words, it is possible to reduce the inertial mass and therefore the gravitational mass of a moving system or object and, consequently, the theoretical possibility is considered that the mechanical energy of a rotating object traversed by a beam of light is transformed into a coordinated acceleration acting on the same object.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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